Grade Level/Course: Algebra 1

Lesson/Unit Plan Name: Arithmetic Sequences

Rationale/Lesson Abstract: Students will be introduced to sequences and learn the characteristic that make sequences arithmetic. In addition, students will write the recursive and explicit formulas by analyzing patterns. Lastly, students will make connections between arithmetic sequences and functions.

Timeframe: Two class periods

Common Core Standard(s): F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Note: the Warm-Up is on page 10.

Instructional Resources/Materials: Warm-Up, Mix and Match Activity Cards

Lesson:

Think-Pair-Share: Can you find a pattern and use it to guess the next term?

A) $7, 10, 13, 16, \ldots$
B) $14, 8, 2, -4, \ldots$
C) $1, 4, 9, 16, \ldots$

TPS Goal: Students notice that the terms in the first sequence are increasing by 3, the terms in the second sequence are decreasing by 6, and the terms in the third are not increasing by a constant.

A) 19 or $a_5 = 19$
B) -10 or $a_5 = -10$
C) 25 or $a_5 = 25$

A sequence is a list or an ordered arrangement of numbers, figures or objects. The members, which are also elements, are called the terms of the sequence. A general sequence can be written as

$$a_1, a_2, a_3, a_4, a_5, a_6, \ldots$$

where a_1 is the first term, a_2 is the second term, and so on. The n^th^ term is denoted as a_n.

Example 1: Find the missing terms in each sequence.

a) $a_1, a_2, a_3, \ldots,$ ____, $a_{125},$ ____, $\ldots$

$a_1, a_2, a_3, \ldots, a_{125-1}, a_{125}, a_{125+1}, \ldots$

$a_1, a_2, a_3, \ldots, a_{124}, a_{125}, a_{126}, \ldots$

b) $a_1, a_2, a_3, \ldots,$ ____, $a_n,$ ____, $\ldots$

$a_1, a_2, a_3, \ldots, a_{n-1}, a_n, a_{n+1}, \ldots$

An arithmetic sequence is a list of numbers in which the difference between two consecutive terms is constant. The common difference is called *d*.

Note: If d > 0, then the terms of the sequence are increasing, and if d < 0, then the terms are decreasing.

Think-Pair-Share: Determine if each sequence is arithmetic. If yes, identify the common difference.

A) $7, 10, 13, 16, \ldots$
B) $14, 8, 2, -4, \ldots$
C) $1, 4, 9, 16, \ldots$

Answers: A) Yes, $d = 3$ B) Yes, $d = -6$ C) No

Example 2: Show that the sequence is arithmetic. Then find the next term in the sequence.

$$7, 10, 13, 16, \ldots$$

Use the definition to show that the sequence is arithmetic: show that the difference between consecutive terms is constant.

$a_2 - a_1$ $= 10 - 7$ $= 3$	$a_3 - a_2$ $= 13 - 10$ $= 3$	$a_4 - a_3$ $= 16 - 13$ $= 3$	…	$a_n - a_{n-1}$ $= ?$

Notice for all the given terms, $a_n - a_{n-1} = 3$. So the difference between consecutive terms is constant and the common difference is 3, or $d = 3$.

Method 1 Use $a_n - a_{n-1} = 3$ to find a_5	Method 2 Rewrite each term in using the *d* and the previous term
$a_5 - a_4 = 3$ $a_5 - 16 = 3$ $a_5 - 16 = 19 - 16$ $a_5 = 19$	$a_1 = 7$ $a_2 = 7 + 3$ $a_3 = 10 + 3$ $a_4 = 13 + 3$ $a_5 = 16 + 3$

$\therefore a_5 = 19$ and the next term in the sequence is 19.

We can continue this pattern to expand the sequence further. When you add *d* to the last known term to get the next, you are using recursion. A recursive formula is a formula that expresses any term a_n in terms of the previous term a_{n-1}. Some examples:

a_6 can be found using a_5: $a_6 = a_5 + 3$ a_7 can be found using a_6: $a_7 = a_6 + 3$ …
a_n can be found using a_{n-1}: $a_n = a_{n-1} + 3$

*Continue the discussion as needed to get students to understand the notation in the recursive formula.

The recursive formula (or rule) for an arithmetic sequence is $a_n = a_{n-1} + d$ where a_1 is given.

Example 3: Write the recursive formula for the sequence $7, 10, 13, 16, \ldots$

The formula depends on the common difference which we already identified from the previous example. Substitute $d = 3$ into $a_n = a_{n-1} + d$ and identify a_1. The recursive formula for this sequence is $a_n = a_{n-1} + 3$ where $a_1 = 7$.

TRY: Find the next term and write the recursive formula for each sequence.

Partner A: $75, 87, 99, \ldots$ $a_4 = 111, \ a_n = a_{n-1} + 12, a_1 = 75$

Partner B: $27, 21, 15, 9, 3, \ldots$ $a_6 = -3, \ a_n = a_{n-1} - 6, a_1 = 27$

Discuss: Why is it necessary to identify a_1 in the recursive formula?

If a_1 is not identified, then the formula represents any sequence who has the same common difference. For example, the sequences $7, 10, 13, 16, \ldots$ and $2, 5, 8, 11, \ldots$ have the same common difference but the recursive formula for the second sequence is $a_n = a_{n-1} + 3$ where $a_1 = 2$.

Example 4: Find the 50th term in the sequence $7, 10, 13, 16, \ldots$

Discuss: We can use the recursive formula repeatedly to obtain desired terms. Or is there another way?

Decompose each term to find a pattern:	Separate each term:
$7, 10, 13, 16, \ldots$ $7, 7+3, 10+3, 13+3, \ldots$ $7, 7+3, 7+3+3, 7+3+3+3, \ldots$ $7, 7+3, 7+3(2), 7+3(3), \ldots$	$a_1 = 7$ $a_2 = 7 + 3(1)$ $a_3 = 7 + 3(2)$ $a_4 = 7 + 3(3)$ $\ldots$ $a_{50} = 7 + 3(49)$ $\therefore a_{50} = 154$

Look for a pattern:

How many groups of 3 are added to 7 in the 50th term?

Answer: 49

Any term in a sequence can be found with an explicit formula, which does not depend on the previous term. An explicit formula is a formula that expresses any term a_n in terms of n, its position in the sequence.

Recall: $f(x) = mx + b$ is a formula, more specifically a function, written in terms of *x*.

Give other examples, if necessary, to build from prior knowledge.

The explicit formula (or rule) for an arithmetic sequence is $a_n = a_1 + (n-1)d$.

Example 5: Write the explicit formula for the sequence. Then find a_{100}.

$$7, 10, 13, 16, \ldots$$

Decompose each term to find a pattern:	Separate each term:
$7, 10, 13, 16, \ldots$ $7,\ 7+3,\ 10+3,\ 13+3,\ \ldots$ $7,\ 7+3,\ 7+3+3,\ 7+3+3+3,\ \ldots$ $7,\ 7+3,\ 7+3(2),\ 7+3(3),\ \ldots$	$a_1 = 7$ $a_2 = 7+3(1)$ $a_3 = 7+3(2)$ $a_4 = 7+3(3)$ $\ldots$ $a_{50} = 7+3(49)$ $a_n = 7+3(n-1)$
Explicit Formula	$a_n = 7+3(n-1)$ or $a_n = 3n+4$
Find a_{100}	$a_n = 7+3(n-1)$ $a_{100} = 7+3(100-1)$ $= 7+3(99)$ $= 7+297$ $= 304$ or $a_n = 3n+4$ $a_{100} = 3(100)+4$ $= 300+4$ $= 304$ $\therefore a_{100} = 304$

TRY: Write the explicit formula for each sequence. Then use it to find a_{30}.

Partner A: $75, 87, 99, \ldots$

$a_n = 75 + (n-1)(12)$
or $a_n = 12n + 63$
$a_{30} = 423$

Partner B: $27, 21, 15, 9, 3, \ldots$

$a_n = 27 + (n-1)(-6)$
or $a_n = -6n + 33$
$a_{30} = -147$

Discussion Questions

How are arithmetic sequences similar to linear functions?

The recursive and explicit formulas are both linear.

How is an arithmetic sequence different from a linear function?

A linear function is continuous whose domain is all real numbers. An arithmetic sequence is a collection of points that are not connected which make it not continuous and the domain is all natural numbers $\{1, 2, 3, 4, ..., n\}$.

Is an arithmetic sequence a function?

Yes, an arithmetic sequence is a function whose domain is all natural numbers $\{1, 2, 3, 4, ..., n\}$.

Therefore, the explicit formula of an arithmetic sequence can be written in function notation.

Summary:

Arithmetic Sequence	Common Difference	Recursive Formula	Explicit Formula	Function Notation
$7, 10, 13, 16,...$	$d=3$	$a_n = a_{n-1} + 3,\ a_1 = 7$	$a_n = 7 + 3(n-1)$ or $a_n = 3n + 4$	$f(n) = 3n + 4$
$75, 87, 99,\ ...$	$d=12$	$a_n = a_{n-1} + 12,\ a_1 = 75$	$a_n = 75 + (n-1)(12)$ or $a_n = 12n + 63$	$f(n) = 12n + 63$
$27, 21, 15, 9, 3,$.	$d=-6$	$a_n = a_{n-1} - 6,\ a_1 = 27$	$a_n = 27 + (n-1)(-6)$ or $a_n = -6n + 33$	$f(n) = -6n + 33$

Mix and Match Activity: Each student receives a card. When prompted, students are to seek the person who has a card that matches their card. Students should be asking appropriate questions using the mathematical language presented in the lesson to find their match. When students find their match, you may have them do an extension activity. Extensions may include, writing about their problem, making a poster/foldable, gallery walk, or reshuffling the cards and repeating the process.

Copy and cut the tables on the next two pages. The corresponding cards are matched for you to use an answer key.

Exit Ticket: Determine if each statement is true or false.

1) The sequence $1, 1, 2, 3, 5, 8, 11,...$ is arithmetic. ◯ True ◯ False

2) The sequence $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2},$ ... is arithmetic. ◯ True ◯ False

3) The recursive formula $a_n = a_{n-1} + 1.2,\ a_1 = 7.1$ represents the sequence $7.1, 8.3, 9.5, 10.7, ...$ ◯ True ◯ False

4) The explicit formula $a_n = -2n + 15$ represents the sequence $15, 13, 11, 9, ...$ ◯ True ◯ False

5) The recursive formula $a_n = a_{n-1} + 8,\ a_1 = 12$ and the explicit formula $a_n = 8n + 4$ represent the same sequence. ◯ True ◯ False

ANSWERS: F, T, T, F, T

Is the sequence arithmetic? $-5, -7, -9, -11, -15, \ldots$ If yes, identify the common difference.	No, the difference between consecutive terms is not constant.
Write the recursive formula for the sequence: $0.5, 1, 1.5, 2, 2.5, \ldots$	$a_n = a_{n-1} + 0.5,\ a_1 = 0.5$
Write the explicit formula for the sequence: $1, 3, 5, 7, 9, \ldots$	$a_n = 1 + (n-1)2$
Does the recursive formula $a_n = a_{n-1} + 5,\ a_1 = 0$ represent the sequence $0, 5, 25, 125, \ldots$?	No, the difference between consecutive terms is not 5.
Does the explicit formula $a_n = 3 + (n-1)4$ represent the sequence $3, 7, 11, 15, \ldots$?	Yes, the first term is 3 and the common difference is 4.
Find the sequence who has the recursive formula $a_n = a_{n-1} + 4,\ a_1 = -9$ and the explicit formula $a_n = 4n - 13$.	$-9, -5, -1, 3, 7, \ldots$
Write the explicit formula for an arithmetic sequence where $d = 2$ and $a_4 = 30$.	$a_n = 24 + (n-1)2$
Identify the first five terms of an arithmetic sequence whose common difference is 10.	$33, 43, 53, 63, 73, \ldots$

Is the sequence arithmetic? $-\frac{5}{3}, -1, -\frac{1}{3}, \frac{1}{3}, 1, \ldots$ If yes, identify the common difference.	Yes, the common difference is $\frac{2}{3}$.
Write the recursive formula for the sequence: $14, 11, 8, 5, 2, \ldots$	$a_n = a_{n-1} - 3,\ a_1 = 14$
Write the explicit formula for the sequence: $65, 51, 37, 23, 9, \ldots$	$a_n = -14n + 79$
Does the recursive formula $a_n = a_{n-1} - 5,\ a_1 = 0$ represent the sequence $0, -5, -10, -15, \ldots$?	Yes, the first term is zero and the common difference is -5.
Does the explicit formula $a_n = 3 + (n-1)4$ represent the sequence $3, 12, 48, \ldots$?	No, the first term is 3 but the common difference is not 4.
Find the sequence who has the recursive formula $a_n = a_{n-1} - 1,\ a_1 = 9$ and the explicit formula $a_n = -n + 10$.	$9, 8, 7, 6, 5, \ldots$
Write the explicit rule for an arithmetic sequence where $d = -2$ and $a_4 = 30$.	$a_n = -2n + 38$.
Identify the first five terms of an arithmetic sequence whose common difference is -10.	$55, 45, 35, 25, 15, \ldots$

Warm-Up

CCSS: F-IF 2.0

Given the linear function $f(x)=2x-5$, indicate whether each statement is true or false.

A) $f(-3)=-11$ ○ True ○ False

B) $f(0)=5$ ○ True ○ False

C) $f\left(\frac{1}{2}\right)=4$ ○ True ○ False

D) $f(x)=-3+2(x-1)$ ○ True ○ False

Identify the correct outputs for the false statements.

CCSS: F-IF 4.0

Graph the linear function $g(x)=x+1$ and complete the statements below.

a) $g(x)<0$ when x_______ .

b) $g(x)=0$ when x_______.

c) $g(x)>0$ when x_______.

Write your own true statement about $g(x)$.

Current:

Suppose a movie theater has 42 rows of seats and there are 29 seats in the first row. Each row after the first row has two more seats than the row before it. Fill in the table below to find the number of seats in each row.

Row Number	Number of Seats
1	
2	
3	
4	
5	
10	
30	
42	

Current continued:

Discuss with a partner how you completed the table. Is there another method?

Warm-Up Solutions

CCSS: F-IF 2.0

Given the linear function $f(x) = 2x - 5$, indicate whether each statement is true or false.

A) $f(-3) = -11$ ○ True

B) $f(0) = 5$ ○ False

C) $f\left(\frac{1}{2}\right) = 4$ ○ False

D) $f(x) = -3 + 2(x - 1)$ ○ True

Corrections: $f(0) = -5$ and $f\left(\frac{1}{2}\right) = -4$

CCSS: F-IF 4.0

Graph the linear function $g(x) = x + 1$ and complete the statements below.

a) $g(x) < 0$ when $x < -1$.

b) $g(x) = 0$ when $x = -1$.

c) $g(x) > 0$ when $x > -1$.

Write your own true statement about $g(x)$.

Sample Answer: $g(0) = 1$

Current:

Suppose a movie theater has 42 rows of seats and there are 29 seats in the first row. Each row after the first row has two more seats than the row before it. Fill in the table below to find the number of seats in each row.

Row Number	Number of Seats
1	29
2	31
3	33
4	35
5	37
10	47
30	87
42	111

Other:

Discuss with a partner how you completed the table. Is there another method?

Sample Answer: Instead of finding the number of seats in row 6-9 to get row 10, understand that you will be adding 5 groups of 2 seats (totaling 10 seats) from row 5 to 10. From row 10 to 30, you will be adding 20 groups of 2 seats (40 seats). And from row 30 to 42, you will be adding 12 groups of 2 seats (24 seats).