X Marks the Spot

When you solve an algebra word problem, the first thing you need to do is translate the problem into mathematical terms. There is usually at least one unknown number in the problem. The letter x is commonly used to represent the unknown number.

Here's how to translate a statement into mathematical terms:

Statement: 19 plus two times a number

19 + 2x

Write the following statements using x, y, and z to represent the unknown numbers. The first one has been done for you.

1. twice a number
   2x

2. a number less three

3. the product of two numbers

4. the sum of three numbers

5. the sum of two numbers less 32

6. 2 times the sum of 3 and a number

7. a number plus twice that number

8. the sum of three times a number and four times another number

9. the difference between 10 and a number

10. the sum of a number, 2 times that number, and 5 times the number

11. the difference between 2 times a number and 5 times another number

12. a number decreased by 20

13. twice a number less 5

14. 42 less 7 times a number

15. a number multiplied by 32

16. 3 less than two times a number

17. the sum of two numbers

18. a number divided by 6

19. the difference between three times a number and two times another number

20. a number squared

21. a number tripled

22. a number plus three times another number

23. 30 minus three times a number

24. the sum of 32 and two times a number

25. 3 times a number less 56

26. a number divided by 4

1   ALGEBRA WORD PROBLEMS

Solving for X

An equation is an algebraic statement that says two expressions are equal. Many equations contain an unknown variable.

Example: x + 10 = 15

The equation above states that x + 10 is equal to 15. To solve it means to find the value of the variable x. To do this, we must get x alone on one side of the equation. We can use the addition rule to do this. The addition rule states that *the same number can be added to or subtracted from both sides of an equation without changing the equation.* To get x alone in the equation, subtract 10 from both side of the equation.

x + 10 - 10 = 15 - 10

Therefore: x = 5

To check your answer, substitute 5 for x.   5 + 10 = 15
                                                                                                15 = 15

Write equations and solve the problems below. Show your work.

1. A number plus 20 equals 25
    What is the number?

2. A number decreased by 10 equals 15.
    What is the number?

3. A number plus 15 equals 40.
    What is the number?

4. A number decreased by 15 equals 35.
    What is the number?

For some equations, we must use the multiplication rule to solve for x. The multiplication rule states that *both sides of an equation may be multiplied or divided by the same non-zero number without changing the equation.*

Example: 4x = 36

$$\frac{4x}{4} = \frac{36}{4}$$ (Divide both sides by 4)

x = 9

Write equations and solve the problems below.

5. Eight times a number equals 72.
    What is the number?

6. Nine times a number equals 18.
    What is the number?

7. Five times a number equals 20.
    What is the number?

8. Ten times a number equals 120.
    What is the number?

9. Three times a number equals 33.
    What is the number?

10. Five times a number equals 30.
    What is the number?

Set and Solve

Use the addition and multiplication rules to solve the problems below.
Show your work.

1. Mary drove 30 miles less than her husband Tom. The total
   distance both of them traveled was 190 miles. Let Tom be x and
   Mary be x – 30. Here is the equation: x + (x – 30) = 190

   Complete the calculations to determine how far Tom drove and
   how far Mary drove.
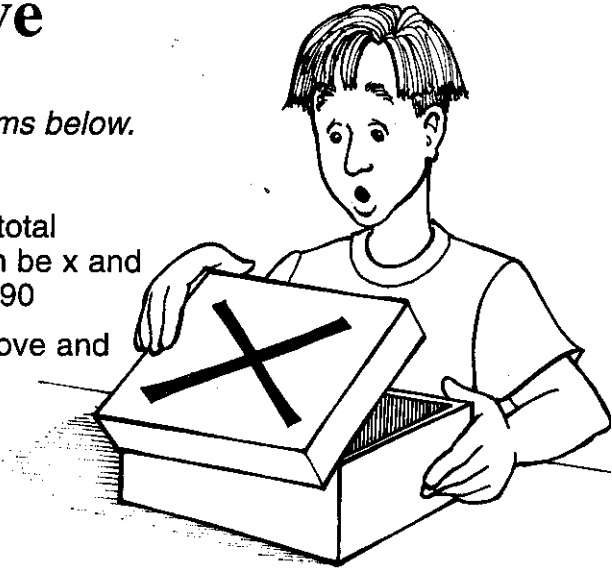
2. The highest grade on a social studies test was 40 points more than the lowest grade. The
   sum of the two grades was 126. Find the lowest grade. Let x = lowest grade and x + 40 be
   the highest grade.

3. A downtown parking lot is 360 feet long. Its length is 4 times its width. How wide is the
   parking lot? Let the width = x.

4. The length of a field is 10 feet longer than 4 times the field's width. The length is 230 feet.
   How wide is the field?

5. On a recent algebra test it took James 20 minutes longer than Manuel to complete the test.
   James took 50 minutes to complete the test. How long did Manuel take?

Money Problems

You can use equations to solve word problems involving money. Here is an example:
Mark has $1.95 in his pocket in nickels and dimes. He has 6 times as many dimes as nickels. How many nickels and how many dimes does he have?

Let x = the number of nickels.
Let 6x = the number of dimes.

To change the values to cents, multiply the number of nickels by 5 and the number of dimes by 10. Then set the equation.

$$5x + 10(6x) = 195$$
$$5x + 60x = 195$$
$$65x = 195$$
$$\frac{65x}{65} = \frac{195}{65}$$
$$x = 3$$

There are 3 nickels and 18 dimes.

Write equations and solve the problems below.

1. Jane has $2.45 in quarters and dimes in her wallet. She has the same number of quarters as dimes. How many of each does she have?

2. Tony has $2.50 in nickels and dimes. He has twice as many dimes as nickels. How many does he have of each?

3. A person gives you coins that total $3.00. They are all quarters and nickels. If you have 5 times as many nickels as quarters, how many nickels and quarters do you have?

4. Fifteen coins, all half-dollars and quarters, total $5.50. How many half-dollars are there? How many quarters are there? (Hint: Let x = number of quarters. Let 15 − x = number of half-dollars.)

Consecutive Integers

You can set up equations to solve problems about consecutive integers. Here is an example:

Find three consecutive integers whose sum is 48.

Let x = the smallest integer.
Let x + 1 = the next integer.
Let x + 2 = the largest integer.

Here is the equation:

$$x + (x + 1) + (x + 2) = 48$$
$$3x + 3 = 48$$
$$3x + 3 - 3 = 48 - 3$$
$$3x = 45$$
$$\frac{3x}{3} = \frac{45}{3}$$
$$x = 15$$
$$x + 1 = 16$$
$$x + 2 = 17$$

You can solve problems about three consecutive odd or even integers by using the values x, x + 2, and x + 4.

1. Find two consecutive integers whose sum is 211.

2. Find four consecutive integers whose sum is 366.

3. Find two consecutive odd integers whose sum is 68.

4. Find four consecutive even integers whose sum is 188.

5. Find five consecutive integers whose sum is 395.
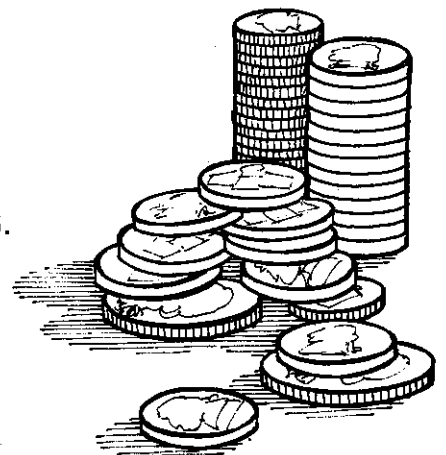
Age Problems

Create equations to solve the following problems involving ages.

1. Jack is twice as old as his sister Tia. Together their ages add up to 21. How old are the two children?

2. Sally is three years more than four times the age of her youngest child. Together their ages add up to 48. How old is Sally? How old is her youngest child?

3. Laura is four years less than twice her son's age. The sum of their ages is 119. How old is Laura? How old is her son?

4. There are four children in a family. They were all born two years apart. The sum of their ages is 84. How old are the children?

5. Paul is 47 years old. His age is two more than three times his daughter's age. How old is his daughter?

6. Moira is three years less than half her mother's age. Her mother is 76 years old. How old is Moira?

On the Farm
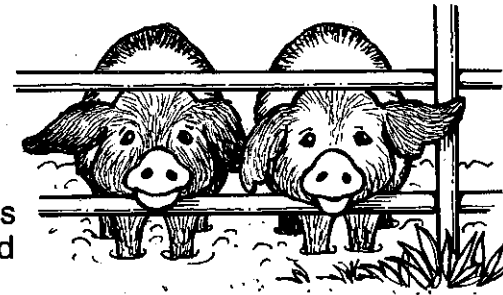
Formulas are an essential part of algebra. They are instructions a person may use to solve a problem. Formulas offer an organized approach to finding a solution. Here is an example:

A person wants to find the total distance around a rectangle (perimeter) that is 10" long and 5" wide. One way to solve this problem would be to add the individual values of all the sides: 10" + 5" + 10" + 5" = 30"

Another way to solve the problem would be to use the formula P = 2L + 2W.

$$P = 2L + 2W$$
$$P = 2(10) + 2(5)$$
$$P = 20 + 10$$
$$P = 30 \text{ inches}$$

Solve the problems below. Show your work.

1. A farmer's field is 400 meters long and 100 meters wide. What is the perimeter of the field in meters?

2. A barn measures 100 feet long and 60 feet wide. What is the perimeter of the barn in feet?

3. A neighboring field is twice as long as the field in problem one, and three times as wide. What is the perimeter of the field?

4. A farmer's field is $\frac{1}{8}$ of a mile square. The fence along its perimeter will have 4 strands of barbed wire. How many total feet of barbed wire are needed? (one mile = 5,280 feet)

5. A farmer is going to fence in his pigs. Their area is 20 feet wide and 30 feet long. The area is right next to the barn, so the farmer only needs to fence three sides of it. If the barn runs the length of the area, how long will the pig fence be?

Name _____

Algebra and Leisure

1. The Arcadia Ballroom needs a new dance floor. A diagram of the ballroom is shown below. How many square feet of maple flooring are required for the new floor?

   Use the formula for the area of a rectangle: $A = LW$

   L

   200'

   80'
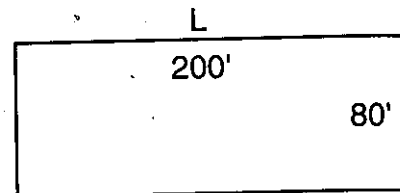
2. A company has been hired to build a round theater. The space for the theater building is 300 feet long on each side. What is the area of the largest circle that will fit in the space?

   Use the formula for the area of a circle: $A = \pi r^2$

   300'

   300'
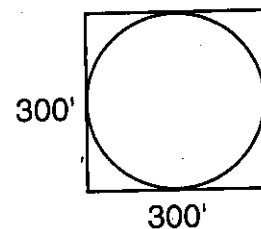
3. A town plans to build a skating rink. A diagram of the proposed rink is shown below. Find the rink's area in square feet.

   First find the area of the rectangle. Next find the area of the two half-circle ends. (Remember that two halves make a whole!) Add the results.

   50'   200'   50'   100'
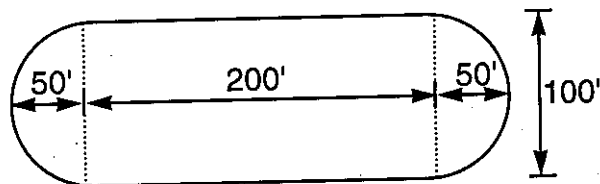
4. A gardener is planning a circular garden. Its diameter is 50 feet. The gardener plans to plant four types of plants. Each type of plant will cover one-fourth of the garden. How many square feet of the garden will be covered with each type of plant? Develop an equation and solve. (Round your answer to the nearest square foot.)

   50'

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ALGEBRA WORD PROBLEMS
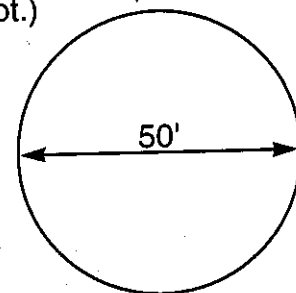
Algebra and Building

A family is finishing a room in their basement. The room is 20 feet long and 14 feet wide. There is a 2' 8"-wide door opening in one wall. Answer the questions below about the family's project.

1. How many square feet are in the room?

   Use the formula A = LW

2. How many linear feet of wall base are needed? Note that no wall base is needed along the door opening.
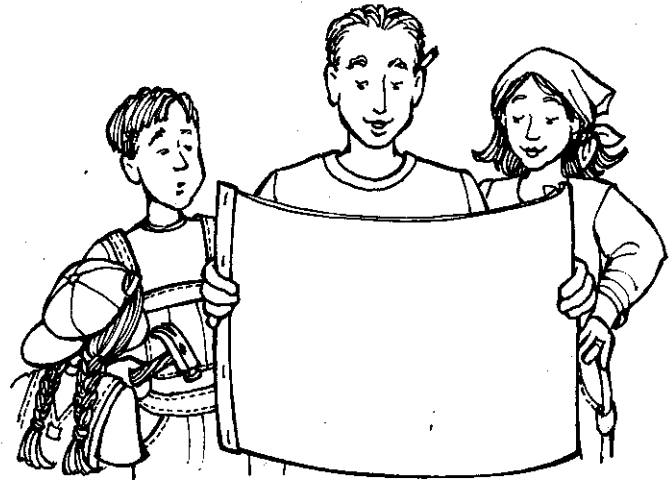
3. Carpet is sold by the square yard (9 square feet). How many square yards of carpet are required for the room? (Round up to the next whole number.)

4. Wall base is purchased in 6' lengths. How many 6' pieces are needed? (Round up to the next whole number.)

5. A standard piece of plywood measures 4' wide and 8' long. How many square feet are in a standard piece of plywood?

6. Refer to problem 5. If $\frac{3}{4}$"-thick plywood costs 40 cents per square foot, what is the cost of 3 pieces of plywood?

7. Sales tax is 5.5%, or $5\frac{1}{2}$ cents on each dollar of cost. How much sales tax is charged on the purchase in problem 6 above? (Round your answer to the nearest cent.)

   Use the formula Tax = Cost x Percent

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Algebra and Building (continued)

Mary Winters is a master gardener. This year she is developing a new rose garden. The garden will be 12' wide and 20' long. Answer the questions below about Mary's garden.

8. How many square feet will the garden be?

20'

12'
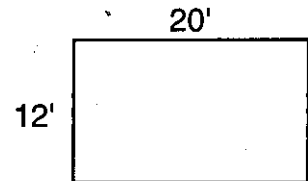
9. Each of Mary's rose bushes requires about 6 square feet of ground area for good growing results. How many rose bushes will she be able to plant in the garden?

10. Mary plans to build a 2'-wide brick walkway around the rose garden. The bricks will be set in a "basket-weave" pattern as shown below. Each square of the pattern is 2' x 2' and contains 18 bricks. How many squares of the pattern are required?

24'

2'

20'

16'   12'
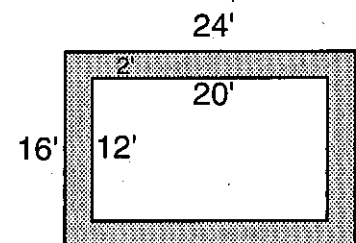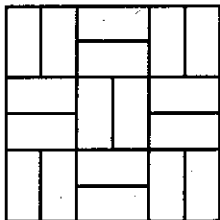
11. How many bricks are required for the walkway?

12. If bricks cost 15 cents ($0.15) each, how much will Mary pay to purchase the bricks?

13. Sales tax is 5.5% on each dollar of cost. How much sales tax will Mary have to pay? (Round your answer to two decimal places.)

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