Arc Length and Area of a Sector

Reporting Category	Polygons and Circles
Topic	Investigating arcs and areas
Primary SOL	G.11 The student will use angles, arcs, chords, tangents, and secants to b) solve real-world problems involving properties of circles; and c) find arc lengths and areas of sectors in circles.
Related SOL	G.14

Materials

- Activity Sheets 1 and 2 (attached)
- Calculators
- Can of pineapple juice (46-fl.oz) and straw (for soda straws extension)
- Duct tape

Vocabulary

circle, chord, diameter, radius, circumference, area of a circle, Pythagorean Theorem, π (pi), height, length, formula, semi-circle, straight segment, distance (earlier grades)

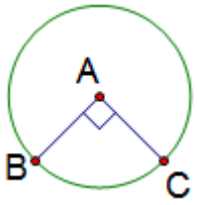
sector, arc, arc measure, arc angle, arc length, central angle, area of a sector (G.11)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students use their calculators to find the lengths of three different ramps that in-line skaters might use for Activity Sheet 1.
2. Review the formula for finding the circumference of a circle, if necessary.
3. Discuss how to find the lengths of various arcs of a circle.
4. Have students use the Pythagorean Theorem to find the length of the third ramp.
5. Hold a class discussion on the following questions:
   - What makes one ramp better than another?
   - Which ramp is safest? Why?
   - Which construction is more challenging? Why?
6. Explain the Cake Problem, as shown on the Activity Sheet 2, to students without handing out the sheets. Make sure everyone understands the problem.
7. Put a 10-inch diameter circle on the overhead projector to model the cake, and ask a volunteer to make an estimate of the placement of the cut that solves the problem.
8. Hand out the activity sheet, and have students work in small groups to solve the problems. Each student should record the solutions on his/her own activity sheet.
9. Have each group present their solutions to the class. Discuss the variations of solutions.
10. Return to the transparency of the cake, and draw the correct solution. How close was the initial estimate?

Assessment

- **Questions**
  - What is the difference between arc length and arc measure?
  - GeoPizza sells 12 inch and 16 inch diameter pizzas. They cut the 12 inch pizza into 6 slices and the 16 inch pizza into 8 slices. Would you get more pizza selecting 3 slices of the 12 inch pizza or 2 slices of the 16 inch pizza? Explain.
  - The length of a minor arc BC of a circle A is 16 pi cm and $\overline{AB} \perp \overline{AC}$. Find the radius of the circle. Justify your answer.



- **Journal/Writing Prompts**
  - Have students complete a journal entry summarizing one of the activities.
  - Write a practical problem and solution using arc length or area of a sector.
  - Describe a formula that could be used to find arc length. Use mathematical vocabulary to explain what each unknown represents.
- **Other**
  - Use the group presentations to assess comprehension.
  - Have students design a ramp and compute its length.

Extensions and Connections (for all students)

- Have students investigate how the Cake Problem changes if a different number of slices are requested.
- Have students find the steepness of the various ramps and use this data to determine the level of difficulty of each ramp.
- Use *The Librarian Who Measured the Earth* by Kathryn Lasky and Kevin Hawkes to investigate Eratosthenes' method for estimating the circumference of the earth.
- Search the web for other descriptions of Eratosthenes' method of estimating the circumference of the Earth.
- Have students do a "Think-Pair-Share" activity in the classroom. Show various pictures of ramps with their measurements. Have students work in pairs to determine which ramps are better for a beginning skater to use. Have them share their results with the class.
- A school track is formed by two straight segments joined by two semicircles. Each straight segment is *l* meters long, and each semicircle is *d* meters in diameter. Write a formula for finding the distance, *D*, around the track.
- Arrange for students to go to the school's baseball diamond, and have them use a tape measure to measure the dimensions of infield. Have them find the area of the infield to determine the amount of dirt needed to cover the infield.
- Arrange for students to visit a local skating park, take measurements, and draw sketches. Have them determine which ramps are the safest after measuring them.
- The book *Around the World in 80 Days* by Jules Verne tells the tale of a voyage around the world by rail and steamer. If an 80-day voyage around the world follows the equator (it didn't in the book) about how long should the voyagers allow to travel from Quito, Ecuador to Libreville, Gabon?
- Research transportation for this part of the voyage via land and water. (No travel by air!)

- Soda Straws: How many straws full of pineapple juice can be taken from a 46-fl. oz. can of juice that is filled to the top? Measure to find the following:
  Diameter of the can: _____ Height of the can: _____
  Diameter of the straw: _____ Length of the straw: _____
  Guess and check. Create an algebraic expression, and use tables.

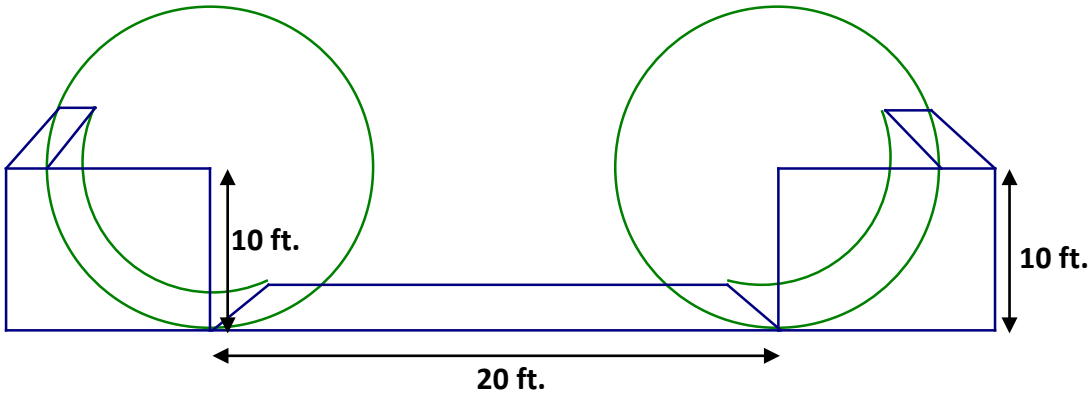
Strategies for Differentiation

- Have students use a dynamic geometry software package to create and draw the ramps. Also have them use the program to measure the distances and the lengths of arcs.
- Have students create a scale model in the classroom with cardboard to replicate the ramps.
- Organize stations to break up and isolate key elements of the lesson. Have students work through the stations in groups to complete the activity.
- Post a math glossary with examples, pictures, and definitions.
- Have students use a graphic organizer to chart information presented in the unit.
- Have students use presentation software to demonstrate their knowledge and understanding of the key terms (e.g., pictures, sound, and motion).
- Have students work in groups to make up their own review materials.
  .

Activity Sheet 1: Problem Solving with Circles

Name _____ **Date** _____

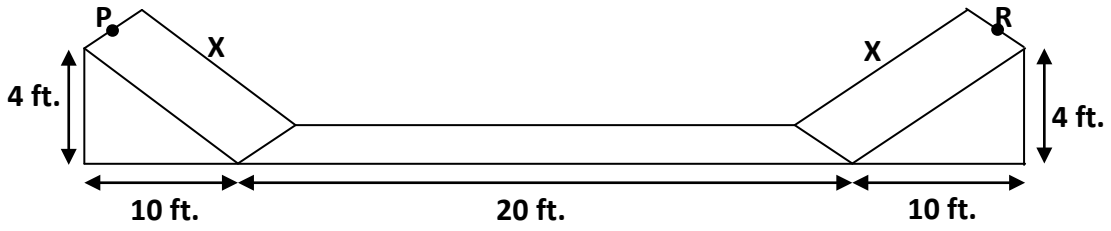
Skateboarding has become a popular sport. The parks department is thinking of constructing ramps in some of the local playgrounds. A "half-pipe" ramp is formed by two quarter-circle ramps, each of which is 10 feet high, plus a flat space 20 feet long between the centers.



1. Find the distance a skater travels from the top of one ramp to the top of the other.

2. Another launch ramp is formed by 2 arcs, each with a central angle of 60 degrees and a radius of 10 ft. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)

3. A third ramp has two straight ramps, each of which is 4 ft. high and 10 ft. long, with a flat space of 20 ft. in between. Find the distance a skater travels from the top of one ramp to the top of the other—from point P to point R. (Hint: Use the Pythagorean Theorem.)



Activity Sheet 2: Cake Problem

Name _____ **Date** _____

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal slices (Figure A).

1. Find the area of each slice of cake.

2. Before you get a chance to serve the cake, 12 more people arrive! So, you decide to cut a concentric circle in the cake so that you will have 24 pieces (Figure B).
3. How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?

4. What is the area of each segment of cake? How much cake will each person receive?

Figure A

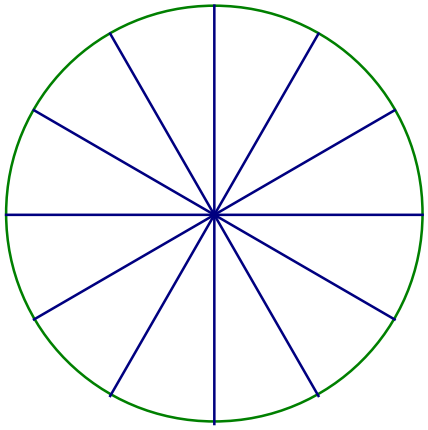


Figure B