HOUSE TRANSFORMATIONS

Student/Class Goal
Gain want to redesign a room in their house and need to make sure the objects can be moved to their new positions.

Outcome *(lesson objective)*
Students will be able to interpret function notation and compare and contrast, as well as perform, translations, rotations, and reflections.

Time Frame
2 hours

Standard *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

NRS EFL
Levels 5-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures	5.6	Word problems	5.25
Calculation	**6.2**	Coordinate system	**6.9**	Problem solving strategies	6.27
Order of operations		Perimeter/area/volume formulas		Solutions analysis	6.28
Compare/order numbers		Graphing two-dimensional figures	5.9	Calculator	
Estimation		Measurement relationships		Math terminology/symbols	6.30
Exponents/radical expressions		Pythagorean theorem		Logical progression	6.31
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	**6.32**
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions		Rounding		Logical terms	5.33
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing	6.18	Data interpretation		Real-life applications	**5.35**
Linear equations		Data displays construction		Independence/range/fluency	5.36
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Vocabulary
Function: A rule that assigns each input to exactly one output.
Transformation: A function that changes an object in form, shape, or appearance.
Translation: A transformation consisting of a constant shift with no rotation or stretching.
Reflection: A transformation that exchanges all points of an object with their mirror image.
Rotation: A transformation that turns an object around a fixed point.

Materials
Furniture layout—Handout
Rulers
SmartPal sleeves/eraser cloths/dry erase markers
Transformations in the Coordinate Plane—Handout
Mira
Patty Paper

Learner Prior Knowledge
Students should be familiar with plotting points on a graph.
Coordinate plane vocabulary.
Function notation, as well as the idea of "inputs" and "outputs".
Basic angles (multiples of 90°).

Instructional Activities
Step 1: Brief review of plotting points and finding coordinates on a graph. Pass out the handout on the furniture layout and have them find the corner points of each piece of furniture. To save time, you can have each student do one piece of furniture and then share answers.

Step 2: We want our students to be able to follow a sequence of steps when solving problems. Whether they know it or not, they probably already sort of do this. We want them to follow Polya's four step process:

1. Understand the problem (*What is the unknown? The data? The conditions?*)
2. Pick a strategy to solve the problem (*Have you seen a similar problem? One with a similar unknown?*)
3. Implement that strategy to come to a solution
4. Review the work and the solution to make sure the solution makes sense in the given context.

After step 4, if there seems to be an error with the solution, students should go back to step 1 and repeat the process until they come to a solution that makes sense.

For the first few lessons, these steps should be discussed and written down so that students can refer to them as a guide when solving problems. During the *I do* steps, your thinking aloud should show you going through all four steps in the process.

Step 3: Functions

If you managed to incorporate the functions concept from "Next Steps" in the previous lesson, you will only need to do a short review on functions. If not, you will need to introduce the students to functions now. They should already be familiar with plotting points and linear equations. Give them the linear equation y=x. This is a line where the y-value is the same as the x-value. Stress that for any x that you "input," you get a certain y value as an "output." This rule that defines that specific "output" is called a function. Now write f(x)=x beneath the y=x equation. Mention that you still "input" values for x. Now, however, the left-hand side looks a little different. We still have an equation, and since the right-hand side has a value, whatever we put in for x, the left-hand side must also have that value. So we can think of "f(x)" as a variable. However, f(x) is more descriptive than y. It tells us we have a function (equation) where we input values for x, since that is the variable noted in parenthesis, and that our output will be just that same value. The function g(x)=x+5, on the other hand, is still a function, the fact that our function is called "g" instead of "f" is irrelevant, but the rule is different. Our input variable is still x, only now our output is found by adding 5 to that variable.

Step 4 : Translations

- (*I do*) A translation should be introduced as synonymous with the word "slide". If we could tangibly slide an object in the coordinate plane to any other position in the plane without changing the orientation of the object, this would be a translation. Whether we take a point or a shape (multiple points that represent the vertices) we add (or subtract) a value from the x-coordinate(s) and y-coordinate(s) of each point. Before starting the first activity, each student should have the handout on the furniture plan (given during step 1), the transformations handout, a ruler, and the SmartPal supplies.
    1. Read the scenario aloud to the class. You will be moving one of the bookcases, let's say the one on the left, and the TV stand. As in the last lesson, you will go through your steps, thinking aloud the whole time. To make sure you understand the problem (step 1 of the Polya process), you need to pick out the key words. You will be moving these two pieces of furniture two spaces closer to that back wall which is at the top of the paper.
    2. For translations, we have two options. We can either visually move the object and then find the coordinates, or use the function notation to find the coordinates and then draw in the correct placement. For the bookcase, do the visual method. For the TV stand, let's use the function notation.
    3. We are sliding the entire bookcase back two units, so that will move it up against the top edge of our graph. Then, find the points of the new position of the bookcase. If we are moving the TV stand "up" two units, then we are adding 2 to each y-coordinate of the current position of the TV stand. Using what was found in step 1, find the new coordinates for the TV stand and then draw it in.
    4. To reflect on this, we look back to the problem. We want the bookcase against the wall, as it is. And we want the "look" of the room to stay the same, so we want the TV stand flush with the front of the bookcase, which it is.
- (*We do*) We're sticking with the same scenario for the student portion. Together, move the other bookcase as well as the coffee table.
    1. The same understanding that happened for moving the first bookcase happens for this bookcase. We want to move it all the way flush with the wall. The coffee table, then, is 5.5 feet away from the TV stand. The first page of the handout says that one space = 1 foot, so we want 5.5 spaces between the two objects.
    2. We have two options to move the objects, visually or using the function, so have the students discuss which they want to use for first the bookcase. Whatever they choose for the bookcase, use the opposite for the coffee table.
    3. Keeping the idea of discussion at the forefront, guide them through moving the bookcase using the chosen method. Then for the coffee table, if visually moving it, make sure they count down 5.5 units from the front of the TV stand or, if using the function method, they need to first figure out how many units the table will move in order to be 5.5 units from the front of the TV stand (so once again they need to find where 5.5 units down from the TV stand will be).
    4. After moving the bookcase, make sure it is against the back wall and the front of the two bookcases and the front of the TV stand form a straight line segment. To check the coffee table, make sure it did not shift left-to-right and make sure there are 5.5 units between the TV stand and the table.
- (*You do*) Have the students move the two end tables and the sofa using whichever method they choose. Walk around to aid them when they need it and to assess their understanding individually.

Step 5 : <u>Reflections</u>

- (*I do*) If a translation=a slide, a reflection=a flip. To do this activity, students will need the miras in addition to previous materials.
  1. Read the second scenario aloud to the class. You will be moving one of the bookcases, let's say the one on the left, and the TV stand. Remember to think aloud the whole time. To make sure you understand the problem (step 1 of the Polya process), you need to pick out the key words. You will be moving these two pieces of furniture to the other side of the room. Looking at the transformation handout, we see that distance is preserved, so the distance from the object to the opposite wall will be the same as it currently is to the wall behind it. It will also be the same distance from the origin as it currently is, just on the other side of the x-axis.
  2. For reflections, we have two options. We can either visually move the object using the mira and then find the coordinates, or use the function notation to find the coordinates and then draw in the correct placement. For the bookcase, do the visual method. For the TV stand, let's use the function notation.
  3. To reflect the bookcase about the x-axis, we want to place the mira on the line of reflection, in this case the x-axis. Looking into the side of the mira that the bookcase is on, we should see a reflection that appears to be on the other side of the paper. We want to trace this reflection so that our bookcase is now reflected across the x-axis. If we are flipping the TV stand across the x-axis then, according to our function, we change the sign of the y-coordinates for the TV stand and then draw it in.
  4. To reflect on this, we look back to the problem. We want the bookcase and TV stand on the other side of the room but the same distance from the new wall behind it, which it is. And we want the "look" of the room to stay the same, so we want the TV stand flush with the front of the bookcase, which it is.
- (*We do*) We're sticking with the same scenario for the student portion. Together, move the other bookcase as well as the coffee table.
  1. The same understanding that happened for moving the first bookcase happens for this bookcase. We want to move it all the way to the other side of the room. The coffee table, however, is a bit different. Since it is closer to the origin, it's reflection won't be nearly as drastic.
  2. We have two options to move the objects, visually or using the function, so have the students discuss which they want to use for first the bookcase. Whatever they choose for the bookcase, use the opposite for the coffee table.
  3. Keeping the idea of discussion at the forefront, guide them through moving the bookcase using the chosen method. Then for the coffee table, if visually moving it, discuss the positioning of the table. Since it is centered over the x-axis, does flipping it really move it? Or, if using the function method, finding the new corner points gives you the same four corner points as before. Do the Harrisons need to move the table to "flip" their room?
  4. After moving the bookcase, make sure it is the correct distance from the wall behind it and the front of the two bookcases and the front of the TV stand form a straight line segment. The coffee table should not have moved.
- (*You do*) Once again, have the students move the sofa and the two end tables using any method they choose.

Step 6: <u>Rotations</u>

- (*I do*) The word to help remember rotations would be "turn." For this activity, students will need patty paper. Give each student at least 5 pieces.
  1. Read the scenario aloud to the class. You will be moving one of the bookcases, let's say the one on the left, and the TV stand. Once again, go through your steps, thinking aloud the whole time. To make sure you understand the problem (step 1 of the Polya process), you need to pick out the key words. You will be moving these two pieces of furniture so that the room is 90° *clockwise* from its current position. This means we have either a rotation of -90° or a rotation of 270°.
  2. For rotations, we have two options. We can either visually move the object using the patty paper and then find the coordinates, or use the function notation to find the coordinates and then draw in the correct placement. For the bookcase, do the visual method. For the TV stand, let's use the function notation.
  3. To rotate the bookcase, take the patty paper so that it overlaps both the entire bookcase and the origin. Trace the bookcase on the patty paper. Then, holding your finger over the origin (it should hold the patty paper to the graph), rotate the patty paper 90° *clockwise.* This shows the new position of the bookcase. For the TV stand, use the function to rotate each point 270°, which means (x,y) becomes (y,-x).
  4. To reflect on this, we look back to the problem. We want the bookcase and the TV stand the same distance from the window as they were the back wall. And we want the "look" of the room to stay the same, so we want the TV stand flush with the front of the bookcase, which it is.
- (*We do*) We're sticking with the same scenario for the student portion. Together, move the other bookcase as well as the coffee table.
  1. The same understanding that happened for moving the first bookcase happens for this bookcase. We want to turn it so that it is on the right side of the room with the back facing the window. We also want to turn the coffee table so

the long edge is facing the TV stand.

2. We have two options to move the objects, visually or using the function, so have the students discuss which they want to use for first the bookcase. Whatever they choose for the bookcase, use the opposite for the coffee table.
3. Keeping the idea of discussion at the forefront, guide them through moving the bookcase using the chosen method. Then for the coffee table, if visually moving it, make sure they hold the patty paper at the origin and turn the patty paper 90° clockwise. Or, if using the function, since a 90° clockwise turn is really a 270° rotation, make sure they choose the right function to use.
4. After moving the bookcase, make sure it is facing the correct direction and the front of the two bookcases and the front of the TV stand form a straight line segment. To check the coffee table, make sure it still is centered over the origin and that the longer side faces the window/TV stand.

- (*You do*) Once again, have the students move the sofa and the two end tables using any method they choose.

Assessment/Evidence (*based on outcome)*

Each of the *you do* steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the *we do* steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student's mastery of the concepts.

To do a final check on each student's grasp of the concepts, have them work individually on the following to turn into you (if there is not sufficient time at the end of the lesson, have them bring it to turn in at the beginning of the next class): Have each student create their own furniture layout for a room in their house. This can be any room in the house, not necessarily the living room as in the lesson. Their room must have at least three pieces of movable furniture. The student must then move one piece of furniture using a translation, another piece using a reflection, and a final piece using a rotation. Have them turn in the following:

1. On graph paper, they should have the initial layout of their room. On that same graph, they should notate the new positions of the three moved pieces of furniture (the markers would come in handy here).
2. A description of each transformation. For example: a translation up 2 units and to the left 3 units, a reflection over the x-axis, or a rotation 270° about the origin.
3. The coordinates of the corners of the pieces of furniture they'll be moving before and after the transformation.

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Lines of reflection other than the two axes, points of rotation other than the origin, and angles of rotation other than the three given.

Technology Integration

http://www.geogebra.org/en/wiki/index.php/Transformations_English

Transformation activities on Geogebra (below)

http://www.geogebra.org/webstart/geogebra.html

Online graphing tool

Purposeful/Transparent

Students want to be able to apply the concepts of translations, rotations, and reflections into a real-world problem. Teachers will model and then guide them in using these concepts with respect to moving objects around the home.

Contextual

Transformations are normally difficult to put into a real-world context for many students. Many do not see the application outside of the coordinate plane. However, moving objects, like furniture, is one way to put this in context. Movers and contractors use the idea of rotations and translations and the fact that length is preserved when moving objects around inside a house to make sure they have enough room. Artists often use all three transformations in works of art. Basketball, pool, and mini golf all use the concept of a reflection and its ability to preserve angles when the ball bounces off the backboard or the railing.

Building Expertise

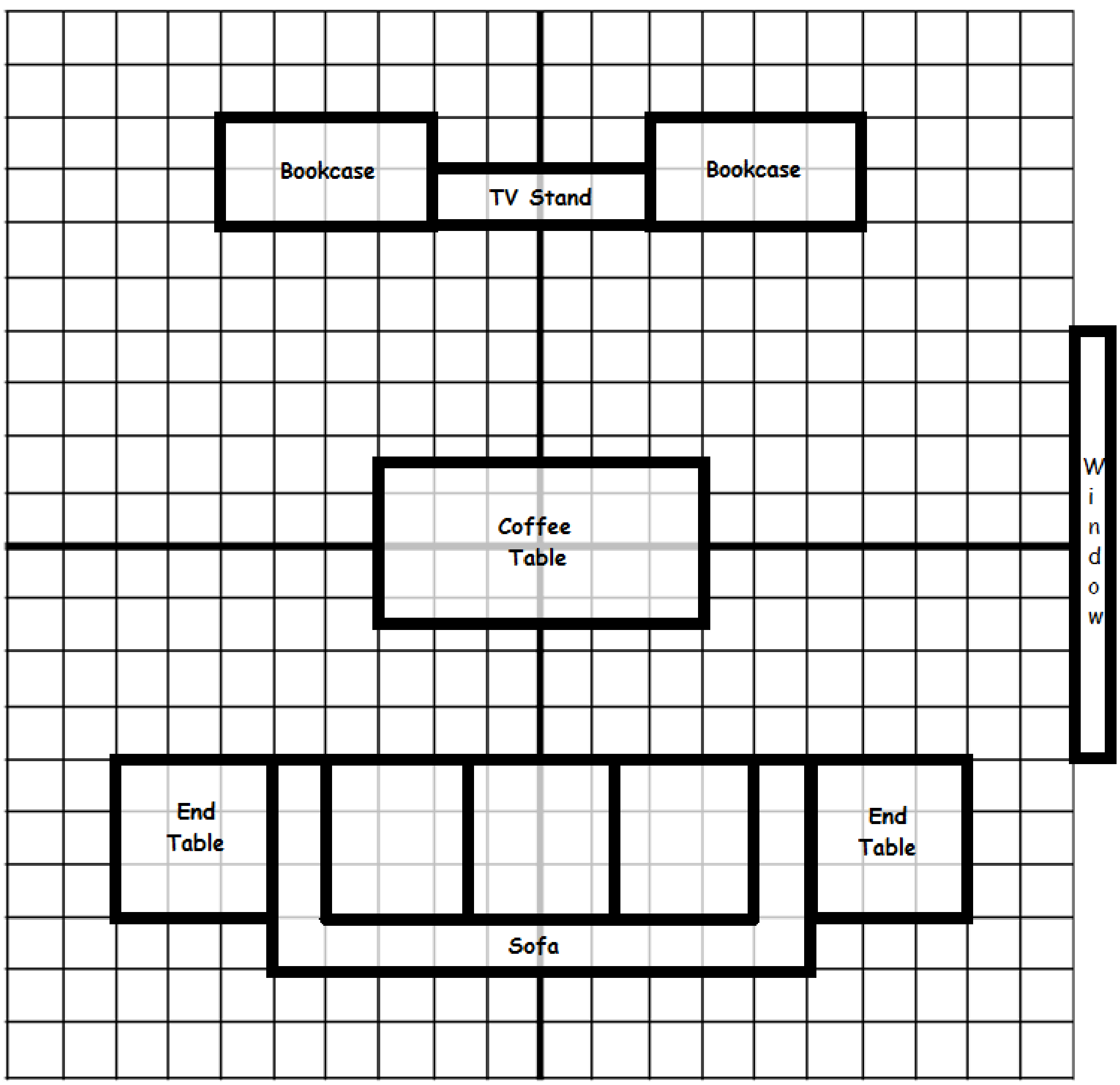
Students will have been familiar with equations and finding points on a graph using an equation prior to this lesson. Now they will be able to recognize function notation and use a function to find points on a graph.

Living Room Furniture Layout (Before)

After moving into a new apartment, the Harrisons decided on the simple layout below for their living room. (***The distance between gridlines is 1 foot.)

Find the four corner points of each piece of furniture in the diagram below:

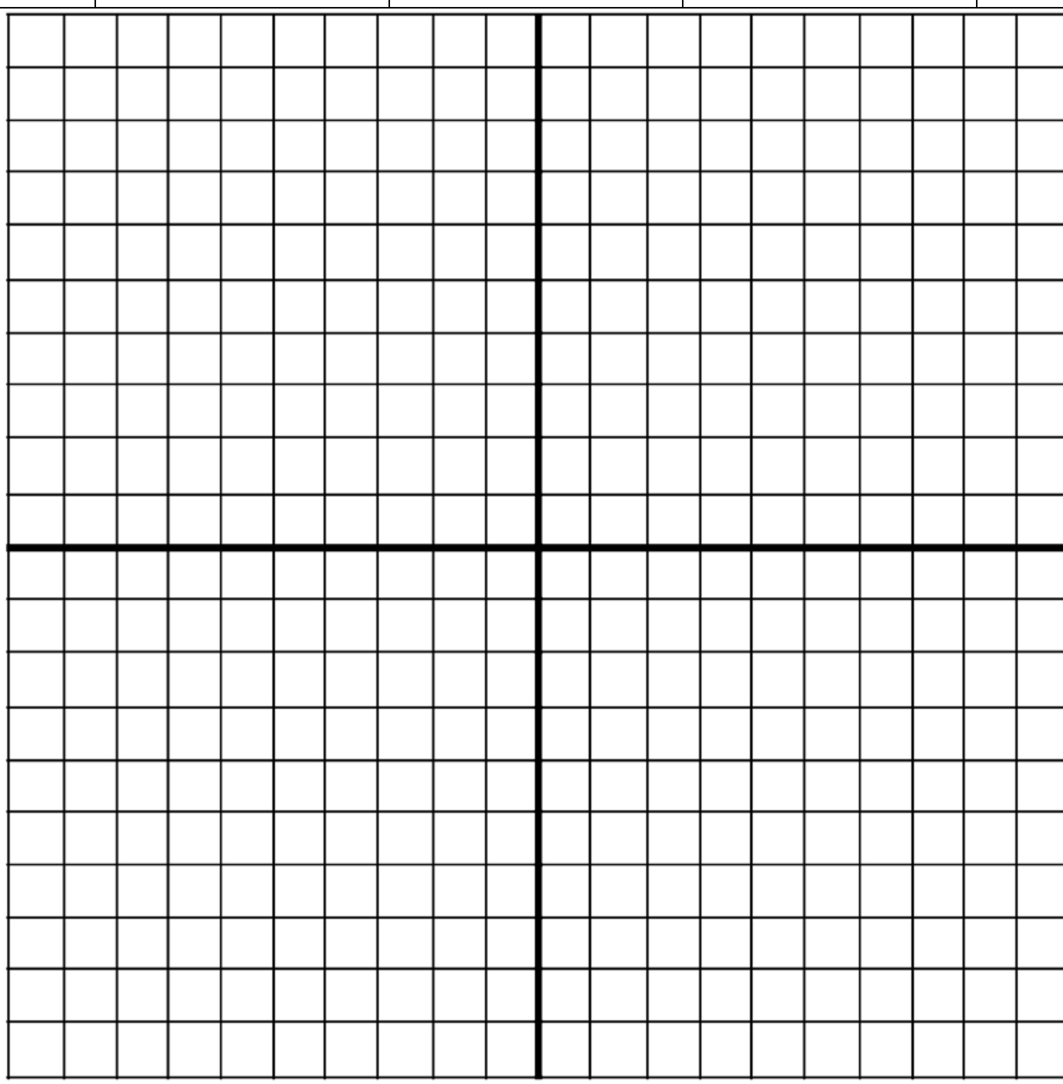
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table				
Sofa				
End Table				
Coffee Table				
Bookcase				
TV Stand				
Bookcase				



Living Room Furniture Layout (Translation)

In order to maximize their free space, the Harrisons decide to move the bookcases and TV stand back 2 feet until the bookcases are against the wall. In additions, they want to have 5.5 feet of space between the front of the TV stand and the coffee table and keep the 2.5 feet of space between the coffee table and sofa. The end tables will move with the sofa so that they stay aligned. Find the four corner points of each piece of furniture in the new setup:

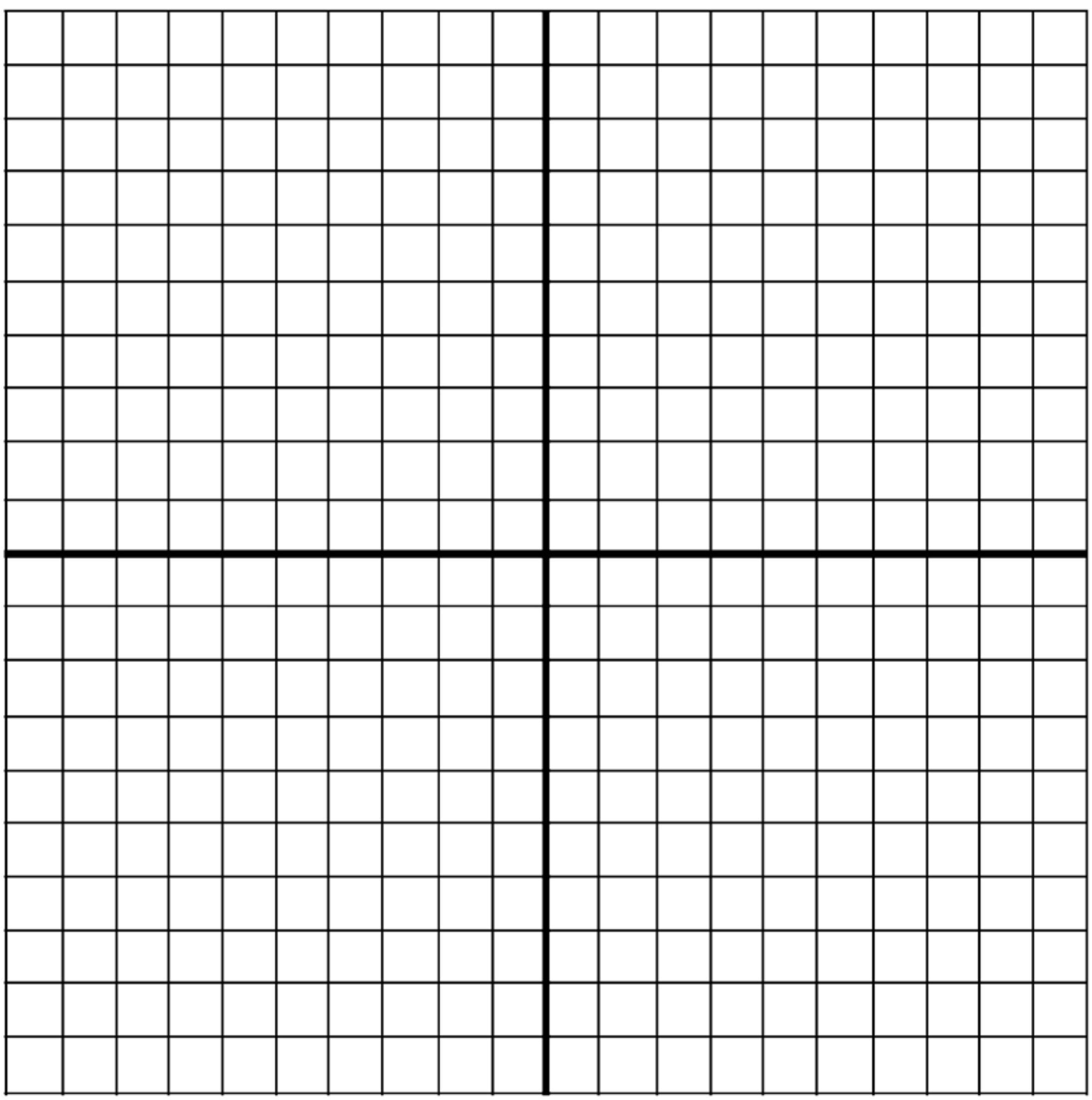
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table				
Sofa				
End Table				
Coffee Table				
Bookcase				
TV Stand				
Bookcase				



Living Room Furniture Layout (Reflection)

From the “before” layout, the Harrisons instead decide that they want to “flip” the room so that everything faces the other direction. They want to reflect every piece of furniture in the room about the center line (the x-axis). Find the four corner points of each piece of furniture in the new layout:

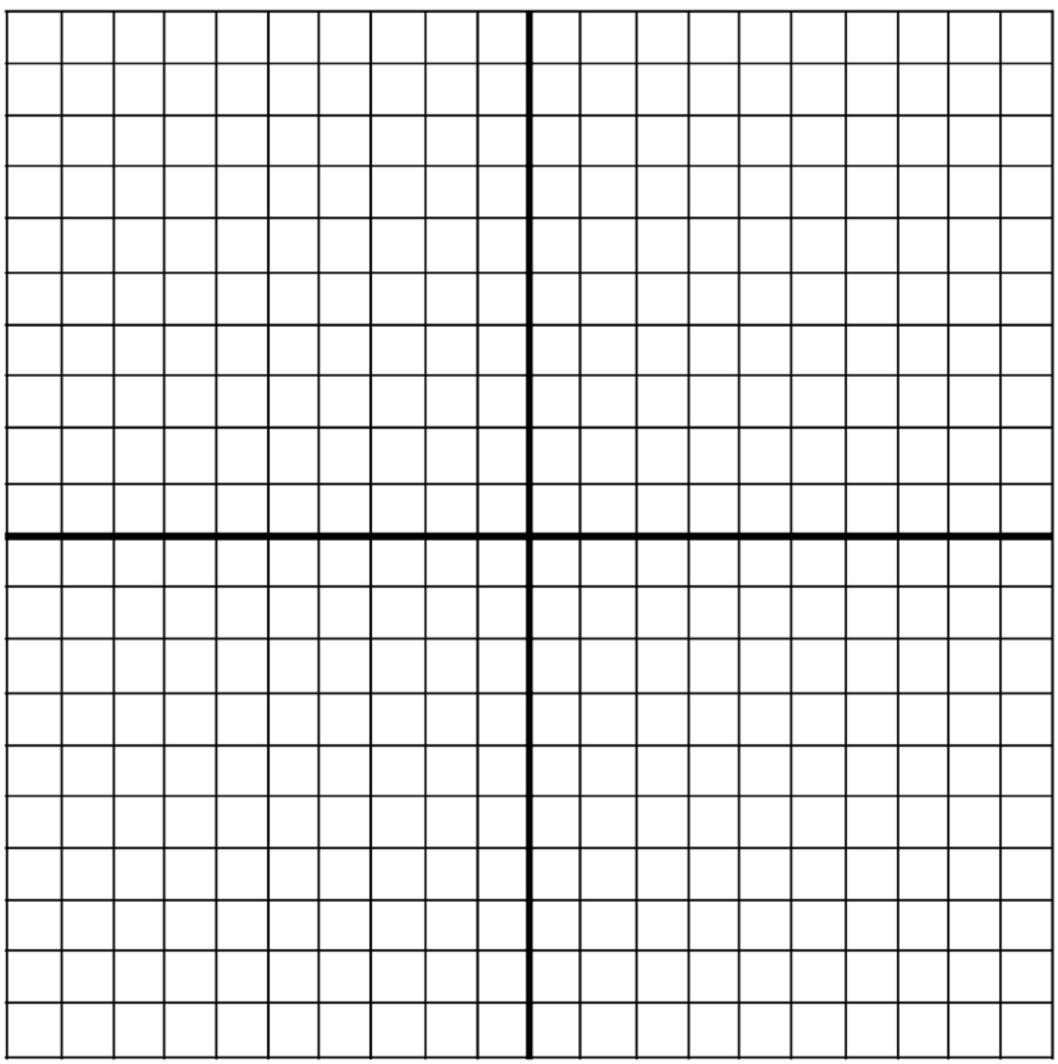
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table				
Sofa				
End Table				
Coffee Table				
Bookcase				
TV Stand				
Bookcase				



Living Room Furniture Layout (Rotation)

After originally having the living room set up as in the “before” layout, the Harrison’s found that the glare from the sun coming through the window made it difficult to watch TV. They decide to rotate the room about the center (the origin) 90 degrees clockwise so that the window is behind the TV. Find the four corner points of each piece of furniture in the new layout:

Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table				
Sofa				
End Table				
Coffee Table				
Bookcase				
TV Stand				
Bookcase				

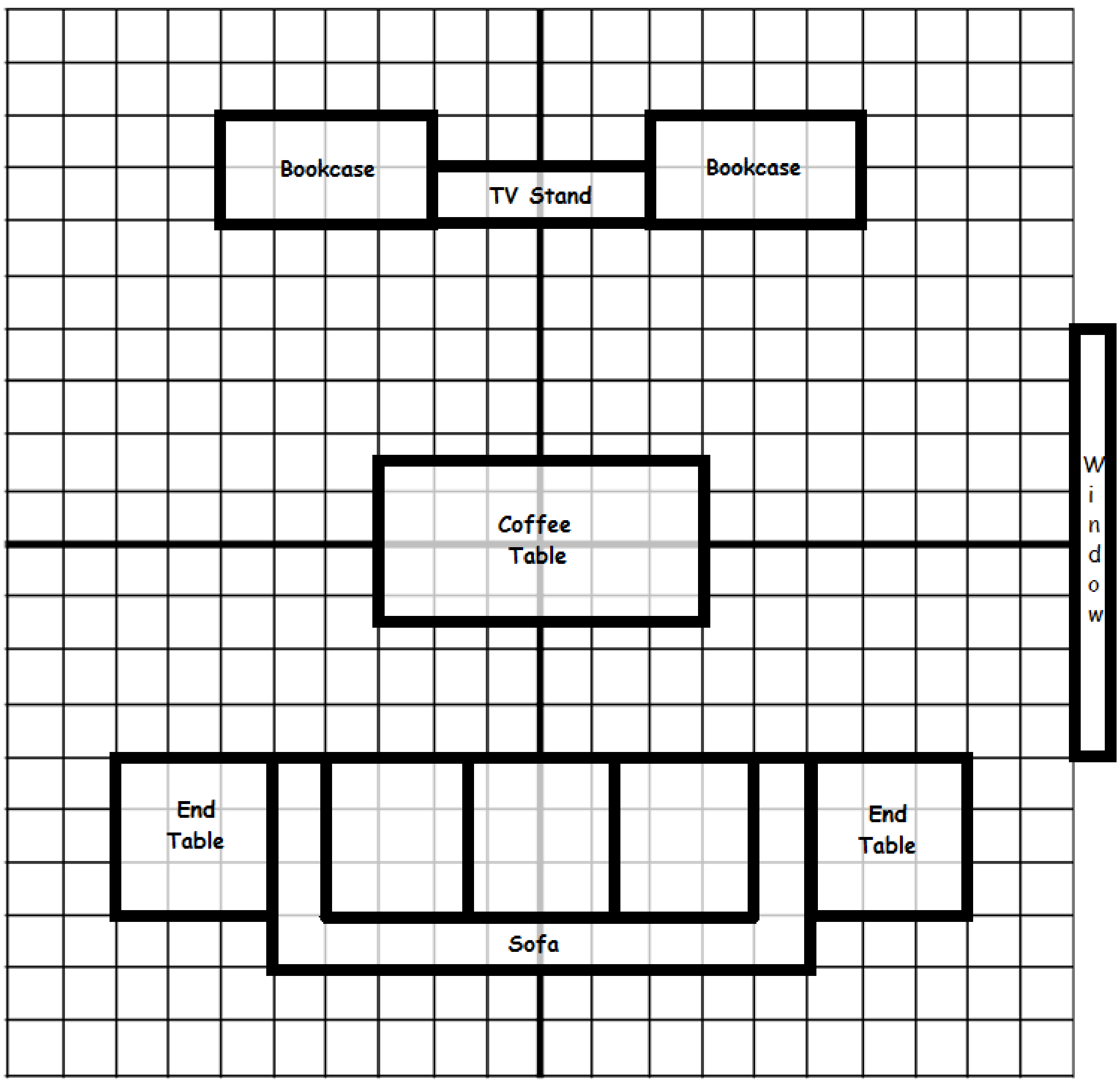


Living Room Furniture Layout (Before)

After moving into a new apartment, the Harrisons decided on the simple layout below for their living room. (***The distance between gridlines is 1 foot.)

Find the four corner points of each piece of furniture in the diagram below:

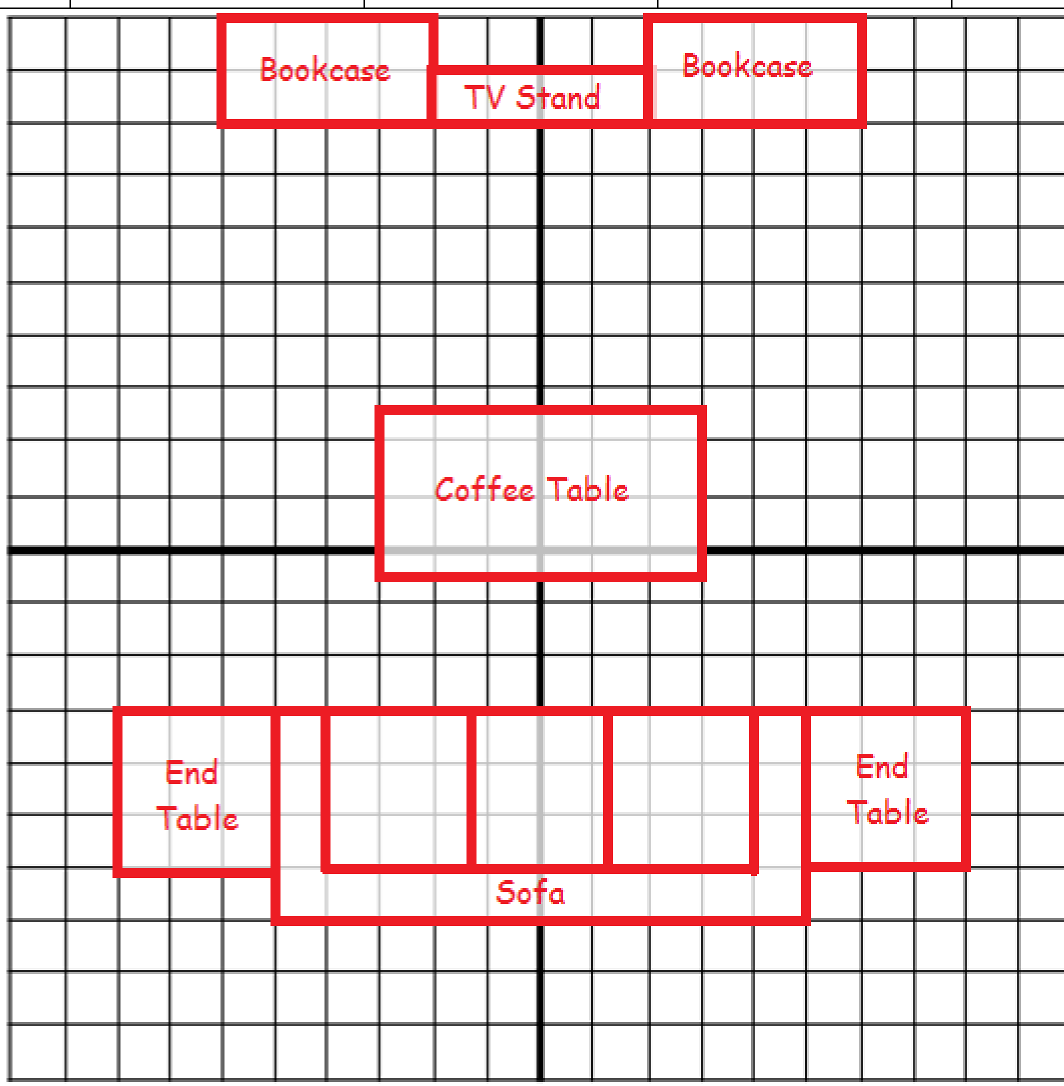
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table	(-8,-4)	(-5,-4)	(-5,-7)	(-8,-7)
Sofa	(-5,-4)	(5,-4)	(5,-8)	(-5,-8)
End Table	(5,-4)	(8,-4)	(8,-7)	(5,-7)
Coffee Table	(-3,1.5)	(3,1.5)	(3,-1.5)	(-3,-1.5)
Bookcase	(-6,8)	(-2,8)	(-2,6)	(-6,6)
TV Stand	(-2,7)	(2,7)	(2,6)	(-2,6)
Bookcase	(2,8)	(6,8)	(6,6)	(2,6)



Living Room Furniture Layout (Translation)

In order to maximize their free space, the Harrisons decide to move the bookcases and TV stand back 2 feet until the bookcases are against the wall. In additions, they want to have 5.5 feet of space between the front of the TV stand and the coffee table and keep the 2.5 feet of space between the coffee table and sofa. The end tables will move with the sofa so that they stay aligned. Find the four corner points of each piece of furniture in the new setup:

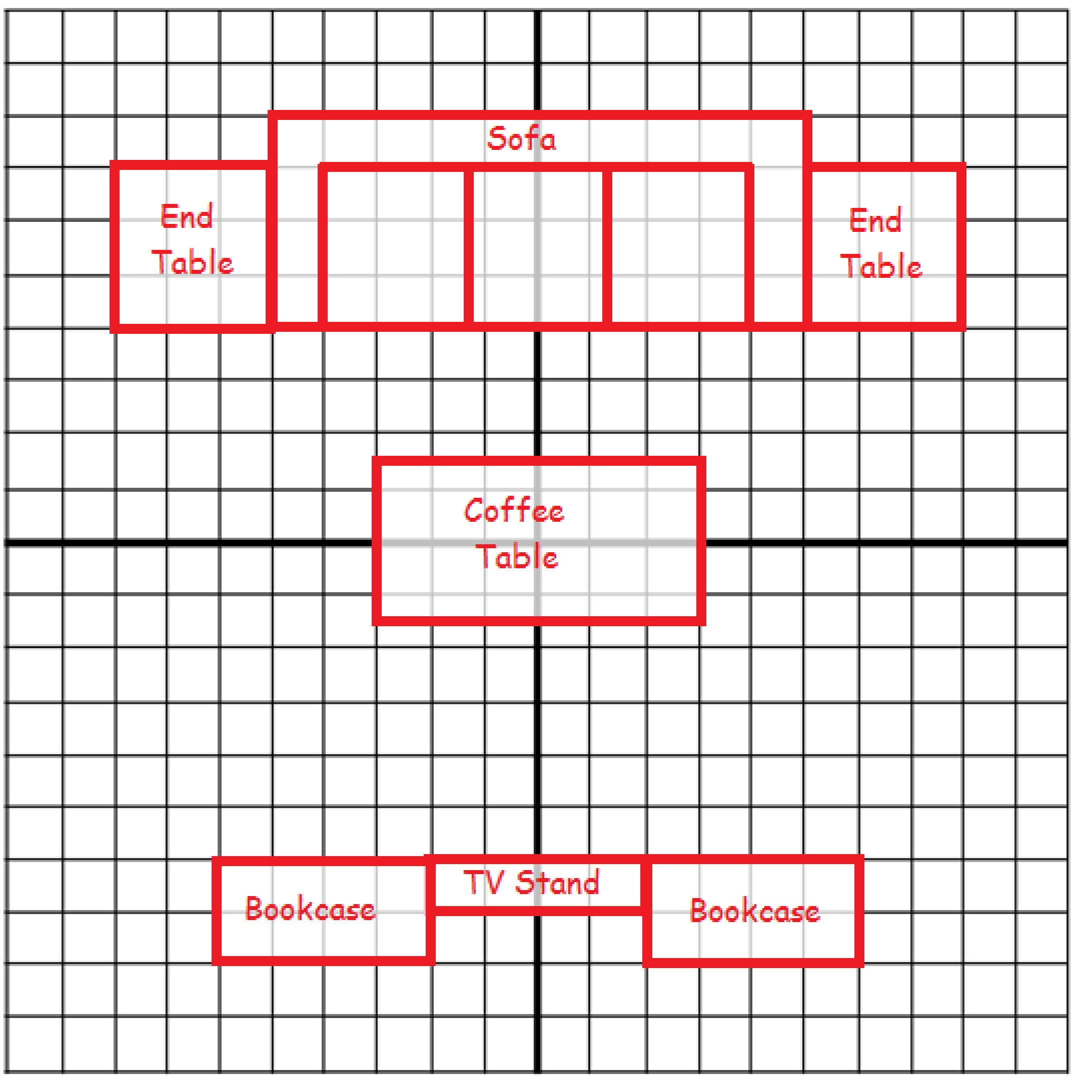
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table	(-8,-3)	(-5,-3)	(-5,-6)	(-8,-6)
Sofa	(-5,-3)	(5,-3)	(5,-7)	(-5,-7)
End Table	(5,-3)	(8,-3)	(8,-6)	(5,-6)
Coffee Table	(-3,2.5)	(3,2.5)	(3,-0.5)	(-3,-0.5)
Bookcase	(-6,10)	(-2,10)	(-2,8)	(-6,8)
TV Stand	(-2,9)	(2,9)	(2,8)	(-2,8)
Bookcase	(2,10)	(6,10)	(6,8)	(2,8)



Living Room Furniture Layout (Reflection)

From the "before" layout, the Harrisons instead decide that they want to "flip" the room so that everything faces the other direction. They want to reflect every piece of furniture in the room about the center line (the x-axis). Find the four corner points of each piece of furniture in the new layout:

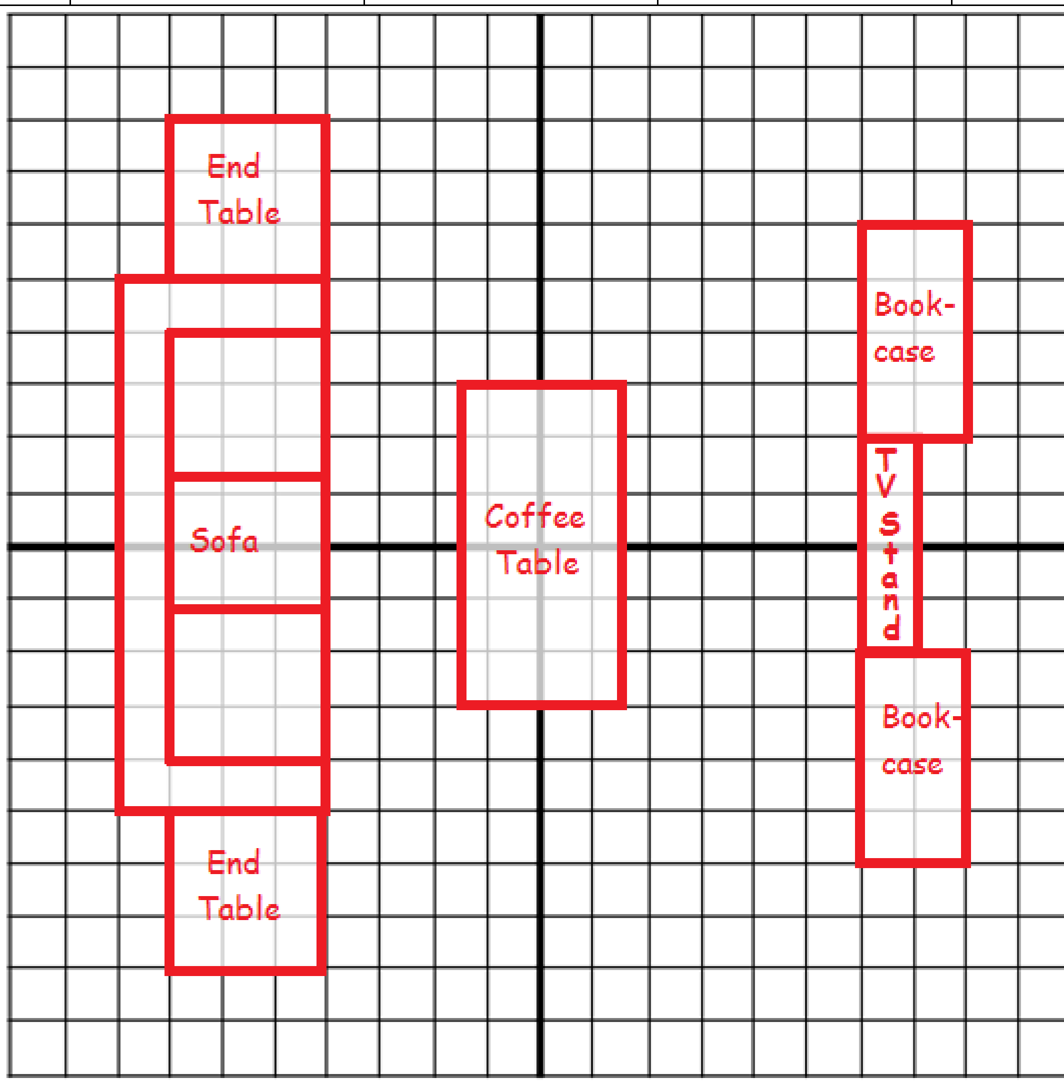
Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table	(-8,4)	(-5,4)	(-5,7)	(-8,7)
Sofa	(-5,4)	(5,4)	(5,8)	(-5,8)
End Table	(5,4)	(8,4)	(8,7)	(5,7)
Coffee Table	(-3,1.5)	(3,1.5)	(3,-1.5)	(-3,-1.5)
Bookcase	(-6,-8)	(-2,-8)	(-2,-6)	(-6,-6)
TV Stand	(-2,-7)	(2,-7)	(2,-6)	(-2,-6)
Bookcase	(2,-8)	(6,-8)	(6,-6)	(2,-6)



Living Room Furniture Layout (Rotation)

After originally having the living room set up as in the "before" layout, the Harrison's found that the glare from the sun coming through the window made it difficult to watch TV. They decide to rotate the room about the center (the origin) 90 degrees clockwise so that the window is behind the TV. Find the four corner points of each piece of furniture in the new layout:

Piece of Furniture	Corner 1 Coordinates	Corner 2 Coordinates	Corner 3 Coordinates	Corner 4 Coordinates
End Table	(-4,8)	(-4,5)	(-7,5)	(-7,8)
Sofa	(-4,5)	(-4,-5)	(-8,-5)	(-8,5)
End Table	(-4,-5)	(-4,-8)	(-7,-8)	(-7,-5)
Coffee Table	(-1.5,3)	(1.5,-3)	(-1.5,-3)	(-1.5,3)
Bookcase	(8,-6)	(8,-2)	(6,2)	(6,6)
TV Stand	(7,2)	(7,-2)	(6,-2)	(6,2)
Bookcase	(8,-2)	(8,-6)	(6,-6)	(6,-2)



Transformations in the Coordinate Plane

In general, a transformation takes an object and moves it to a new position. If we move a point P, we typically label the point as P' after the transformation.

Translations

- A translation "slides" an object to a new position in the coordinate plane.
- Translations preserve the following properties:
  - Distance: Lengths in the original are preserved in the translation. (For example: A side length in equilateral triangle ABC is 5. When we translate this, we will now have equilateral triangle A'B'C' with side length 5.)
  - Angle measures
  - Parallel lines will still be parallel if translated.
  - Midpoints
  - Co-linearity
  - Orientation: The order of labeling is preserved.
- Translations can be written as functions where T(P)=P'. If we let $T_{a,b}(x,y)$ be the output of a translation of the point (*x,y*) *a* units in the *x*-direction and *b* units in the *y*-direction, we arrive at the new point (*x+a,y+b).* Thus, the translation function is:

$$T_{a,b}(x,y) = (x+a, y+b)$$

Reflections

- A reflection is a "flip" or a mirror image of an object over a line in the coordinate plane.
- The distance from the object being reflected to the line of reflection is the same as the distance from the reflected object to the line of reflection.
- Reflections preserve the following properties:
  - Distance
  - Angle measures
  - Parallel lines will still be parallel if translated.
  - Midpoints
  - Co-linearity
- It is important to note that while the two objects will be congruent, the labeling order is *not* preserved in reflections. As this is a mirror image, the labeling order is reversed.
- The two most common lines of reflection in the coordinate plane are the *x-axis* and the *y-axis.*
- As with translations, reflections can be written as functions where r(P)=P'.
  - Let us first consider a reflection over the *x-axis*. In this situation, we are taking an object and flipping it vertically. We make no changes to the *x*-coordinate of the

point, however we move the point to the other side of the *x-axis* so it is the same distance from the axis as the original point. Thus:

$$r_{x-axis}(x,y) = (x,-y)$$

- For a reflection over the *y-axis.* In this situation, we are taking an object and flipping it horizontally. We make no changes to the *y*-coordinate of the point, however we move the point to the other side of the *y-axis* so it is the same distance from the axis as the original point. Thus:

$$r_{y-axis}(x,y) = (-x,y)$$

Rotations

- A rotation is a "turning" of an object about a point of rotation in the coordinate plane.
- The angle formed between the original point and its rotated image is the angle of rotation. (Note: if the object rotated is a shape, the angle of rotation will be the same between all corresponding points—the original point and its rotated image.)
- Angles measured in the counter-clockwise direction are positive while angles measured in the clockwise direction are negative.
- Rotations preserve the following properties:
  - Distance
  - Angle measures
  - Parallel lines will still be parallel if translated.
  - Midpoints
  - Co-linearity
  - Labeling order is preserved.
- The most common point of rotation is the origin and the three most common angles of rotation in the coordinate plane are *90°, 180°,* and *270°.*
- As with translations, reflections can be written as functions where R(P)=P'. For the following three scenarios, the point of rotation in the origin.
  - Let us first consider a *90°* rotation:

$$R_{90^\circ}(x,y) = (-y,x)$$

  - For a rotation of *180*°:

$$R_{180^\circ}(x,y) = (-x,-y)$$

  - For a rotation of *270*°:

$$R_{270^\circ}(x,y) = (y,-x)$$